Notes on Deriving the Lorentz Transformation from Fundamental Effects

In the first 4 exercises you proved the following:

1. Perpendicular Lengths are Measured to be the same
   1. Length measurements of an object made in two different inertial frames of reference will agree if the length being measured is perpendicular to the relative velocity between the frames.
2. Time Dilation
   1. If a single clock at rest in an inertial frame measures the time between two events *at the fixed location of the clock,* Δtp then the time measured between the same two events as measured in another inertial frame Δt’ are related by *the time dilation formula*:

1. Length Contraction
   1. If the length of an object that is at rest in an inertial frame is measured to be Lp *and* the length of the object is measured in another inertial frame of reference to be L’; where the second frame is moving with relative speed v parallel to the measured length then the two lengths are related by the length contraction formula:
2. De-synchronization
   1. If two events happen at the same time in an inertial frame but are separated in space by a distance Δx, then, in another inertial frame of reference moving with speed v and along the line of the spatial separation, the two events no longer simultaneous but are separated in time by

The objective of the 5th Exercise is to synthesize these 4 effects into a simple and complete translation scheme between position and time coordinates for the same event measured in two different inertial frames. The answer can be expressed as a linear transformation called a *Lorentz Transformation*. First off we make some simplifying assumptions; namely, that the two inertial frames have erected coordinate systems with parallel axes and that the relative velocity between the two frames is along the common x axes ( with the ‘prime’ frame measured to be moving in the +x direction with speed v in the un-primed frame) We further assume that the origins of the two frames coincide and are assigned a time t=t’=0. Generally we seek four expressions:

t’= t’(t, x, y z)

x’ = x’(t, x, y, z)

y’=y’(t, x, y, z)

z’=z’(t, x, y, z)

where t, x, y, z are the coordinates assigned to an event in one frame and t’,x’,y’,z’ are the coordinates assigned to the same event in the other frame.

Because the relative velocity is along the x axes, we can quickly conclude from the invariance of perpendicular lengths that for any event

(1) z’=z

(2) y’=y

Your problem is to use what we have already established about length contraction, time dilation, and de-synchronization to determine the remaining relationships:

(3) x’=x’(x,t)

(4) t’=t’(x,t)

*Careful thought* about the relationship between time and space coordinate assignment (measurement) and the results you have already discovered about proper vs non-proper length and proper vs non-proper time measurements and de-synchronization are required for a direct derivation. The most economical derivation uses the symmetry of inertial frames and length contraction alone to derive the remaining critical parts (3) and (4) of the Lorentz Transformation.